```
nx = 21;
Δ = (4 - 1) / (nx - 1)
```

$$\frac{3}{20}$$

```
x[i_] := 1 + Δ (i - 1)
xn = Table[x[i] , {i, 1, nx}]
```

$$\left\{1, \frac{23}{20}, \frac{13}{10}, \frac{29}{20}, \frac{8}{5}, \frac{7}{4}, \frac{19}{10}, \frac{41}{20}, \frac{11}{5}, \frac{47}{20}, \frac{5}{2}, \frac{53}{20}, \frac{14}{5}, \frac{59}{20}, \frac{31}{10}, \frac{13}{4}, \frac{17}{5}, \frac{71}{20}, \frac{37}{10}, \frac{77}{20}, 4\right\}$$

```
matA[i_, j_] := Which[
  i == j, -2,
  i == j - 1, 1,
  i == j + 1, 1,
  i ≤ j - 2, 0,
  i ≥ j + 2, 0
 ]
listmatA = Table[matA[i, j], {i, 1, nx}, {j, 1, nx}];
MatrixForm[%]
```

$$\begin{pmatrix}
-2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2
\end{pmatrix}$$

```
listmatA[[1, 1]] = 1; (* Boundary Condition *)
listmatA[[1, 2]] = -1;
listmatA[[nx, nx - 1]] = 0;
listmatA[[nx, nx]] = 1;
```

```
listmatA // MatrixForm
```

$$\begin{pmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

```
listvecC = Table[Which[i == 1 || i == nx, 0, 1 < i < nx, 6 xn[[i]] Δ^2], {i, 1, nx}]
```

$$\left\{0, \frac{621}{4000}, \frac{351}{2000}, \frac{783}{4000}, \frac{27}{125}, \frac{189}{800}, \frac{513}{2000}, \frac{1107}{4000}, \frac{297}{1000}, \frac{1269}{4000}, \right.$$
$$\left. \frac{27}{80}, \frac{1431}{4000}, \frac{189}{500}, \frac{1593}{4000}, \frac{837}{2000}, \frac{351}{800}, \frac{459}{1000}, \frac{1917}{4000}, \frac{999}{2000}, \frac{2079}{4000}, 0\right\}$$

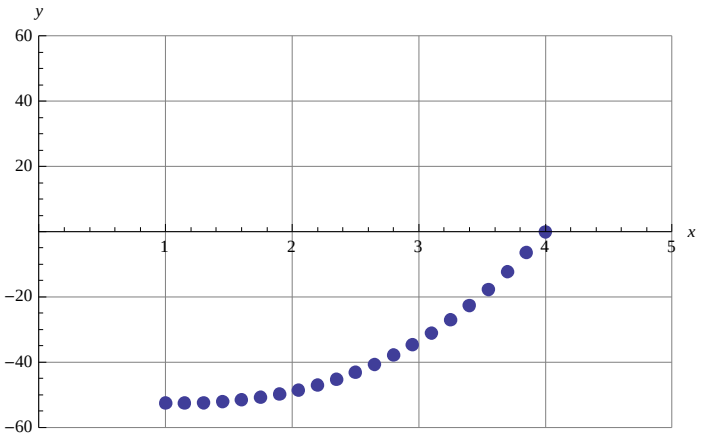
```
vecy = Inverse[listmatA].listvecC
```

$$\left\{-\frac{21\,033}{400}, -\frac{21\,033}{400}, -\frac{209\,709}{4000}, -\frac{104\,193}{2000}, -\frac{5157}{100}, -\frac{20\,331}{400}, \right.$$
$$-\frac{39\,879}{800}, -\frac{97\,227}{2000}, -\frac{94\,203}{2000}, -\frac{18\,117}{400}, -\frac{34\,533}{800}, -\frac{16\,281}{400}, -\frac{37\,881}{1000},$$
$$\left. -\frac{69\,363}{2000}, -\frac{24\,867}{800}, -\frac{10\,827}{400}, -\frac{1809}{80}, -\frac{35\,397}{2000}, -\frac{49\,221}{4000}, -\frac{513}{80}, 0\right\}$$

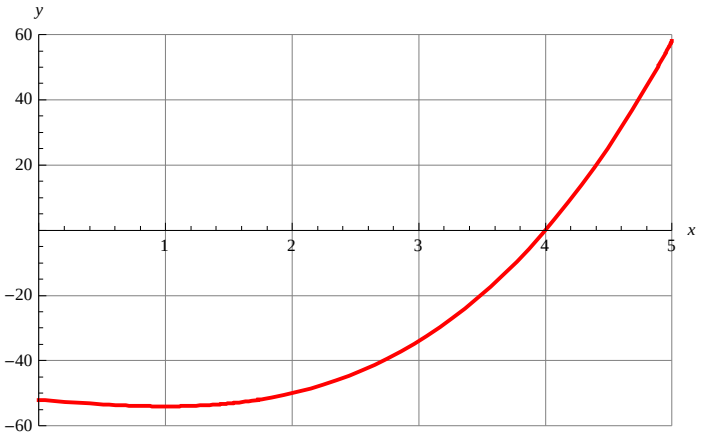
```
data = Table[{xn[[i]], vecy[[i]]}, {i, 1, nx}]
```

$$\left\{\left\{1, -\frac{21\,033}{400}\right\}, \left\{\frac{23}{20}, -\frac{21\,033}{400}\right\}, \left\{\frac{13}{10}, -\frac{209\,709}{4000}\right\}, \left\{\frac{29}{20}, -\frac{104\,193}{2000}\right\}, \left\{\frac{8}{5}, -\frac{5157}{100}\right\},\right.$$
$$\left\{\frac{7}{4}, -\frac{20\,331}{400}\right\}, \left\{\frac{19}{10}, -\frac{39\,879}{800}\right\}, \left\{\frac{41}{20}, -\frac{97\,227}{2000}\right\}, \left\{\frac{11}{5}, -\frac{94\,203}{2000}\right\}, \left\{\frac{47}{20}, -\frac{18\,117}{400}\right\},$$
$$\left\{\frac{5}{2}, -\frac{34\,533}{800}\right\}, \left\{\frac{53}{20}, -\frac{16\,281}{400}\right\}, \left\{\frac{14}{5}, -\frac{37\,881}{1000}\right\}, \left\{\frac{59}{20}, -\frac{69\,363}{2000}\right\}, \left\{\frac{31}{10}, -\frac{24\,867}{800}\right\},$$
$$\left.\left\{\frac{13}{4}, -\frac{10\,827}{400}\right\}, \left\{\frac{17}{5}, -\frac{1809}{80}\right\}, \left\{\frac{71}{20}, -\frac{35\,397}{2000}\right\}, \left\{\frac{37}{10}, -\frac{49\,221}{4000}\right\}, \left\{\frac{77}{20}, -\frac{513}{80}\right\}, \{4, 0\}\right\}$$

```
g0 = ListPlot[data, GridLines -> Automatic, AxesLabel → {x, y},
  PlotStyle → PointSize[Large], PlotRange → {{0, 5}, {-60, 60}}]
```




```
g1 = Plot[x^3 - 3 x - 52, {x, 0, 5}, AxesLabel → {x, y},
  PlotStyle → {Red, Thick}, GridLines -> Automatic, PlotRange → {{0, 5}, {-60, 60}}]
```




```
Show[g1, g0]
```