```
nx = 16;
Δ = (4 - 1) / (nx - 1)
```

$$\frac{1}{5}$$

```
x[i_] := 1 + Δ (i - 1)
xn = Table[x[i] , {i, 1, nx}]
```

$$\left\{1, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}, 2, \frac{11}{5}, \frac{12}{5}, \frac{13}{5}, \frac{14}{5}, 3, \frac{16}{5}, \frac{17}{5}, \frac{18}{5}, \frac{19}{5}, 4\right\}$$

```
Table[a11[i] = 1 / (x[i + 1] - x[i]), {i, 1, nx - 1}];
Table[a12[i] = -1 / (x[i + 1] - x[i]), {i, 1, nx - 1}];
Table[a21[i] = a12[i], {i, 1, nx - 1}];
Table[a22[i] = a11[i], {i, 1, nx - 1}];
Table[c1[i] = (-2 (x[i + 1]^3 - x[i]^3) + 3 x[i + 1] (x[i + 1]^2 - x[i]^2)) / (x[i + 1] - x[i]),
 {i, 1, nx - 1}]
Table[c2[i] = (2 (x[i + 1]^3 - x[i]^3) - 3 x[i] (x[i + 1]^2 - x[i]^2)) / (x[i + 1] - x[i]),
 {i, 1, nx - 1}]
```

$$\left\{\frac{16}{25}, \frac{19}{25}, \frac{22}{25}, 1, \frac{28}{25}, \frac{31}{25}, \frac{34}{25}, \frac{37}{25}, \frac{8}{5}, \frac{43}{25}, \frac{46}{25}, \frac{49}{25}, \frac{52}{25}, \frac{11}{5}, \frac{58}{25}\right\}$$

$$\left\{\frac{17}{25}, \frac{4}{5}, \frac{23}{25}, \frac{26}{25}, \frac{29}{25}, \frac{32}{25}, \frac{7}{5}, \frac{38}{25}, \frac{41}{25}, \frac{44}{25}, \frac{47}{25}, 2, \frac{53}{25}, \frac{56}{25}, \frac{59}{25}\right\}$$

```
For[i = 1, i ≤ nx, i++,
 For[j = 1, j ≤ nx, j++,
  Which[
   j == 1 && i == 1, matA[i, j] = a11[j],
   j == nx && i == nx, matA[i, j] = a22[j - 1],
   j == i, matA[i, j] = a22[j - 1] + a11[j],
   j == i + 1, matA[i, j] = a12[j - 1],
   j == i - 1, matA[i, j] = a21[j],
   j ≥ i + 2, matA[i, j] = 0,
   j ≤ i - 2, matA[i, j] = 0
  ]
 ]
]
Table[matA[i, j], {i, 1, nx}, {j, 1, nx}];
MatrixForm[%]
```

$$\begin{pmatrix}
5 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 5
\end{pmatrix}$$

```
For[i = 1, i ≤ nx, i++,
 Which[
  i == 1, vecC[i] = -c1[i],
  i ≥ 2 && i ≤ nx - 1, vecC[i] = -c2[i - 1] - c1[i],
  i == nx, vecC[i] = -c2[i - 1]
 ]
]
```

```
Table[vecC[i], {i, 1, nx}] // MatrixForm;
```

```
For[i = 1, i ≤ nx, i++,
 For[j = 1, j ≤ nx, j++,
  matAb[i, j] = matA[i, j]
 ]
]
matAb[1, 1] = 1; (* Boundary Condition*)
matAb[1, 2] = -1;
matAb[nx - 1, nx] = 0;
matAb[nx, nx - 1] = 0;
matAb[nx, nx] = 1;
```

```
listmatAb = Table[matAb[i, j], {i, 1, nx}, {j, 1, nx}];
```

```
MatrixForm[%]
```

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Table[vecCb[i] = vecC[i], {i, 1, nx}]; (* Boundary Condition*)
vecCb[1] = 0;
vecCb[nx] = 0;
```

```
listvecCb = Table[vecCb[i], {i, 1, nx}]
```

$$\left\{0, -\frac{36}{25}, -\frac{42}{25}, -\frac{48}{25}, -\frac{54}{25}, -\frac{12}{5}, -\frac{66}{25}, -\frac{72}{25}, -\frac{78}{25}, -\frac{84}{25}, -\frac{18}{5}, -\frac{96}{25}, -\frac{102}{25}, -\frac{108}{25}, -\frac{114}{25}, 0\right\}$$

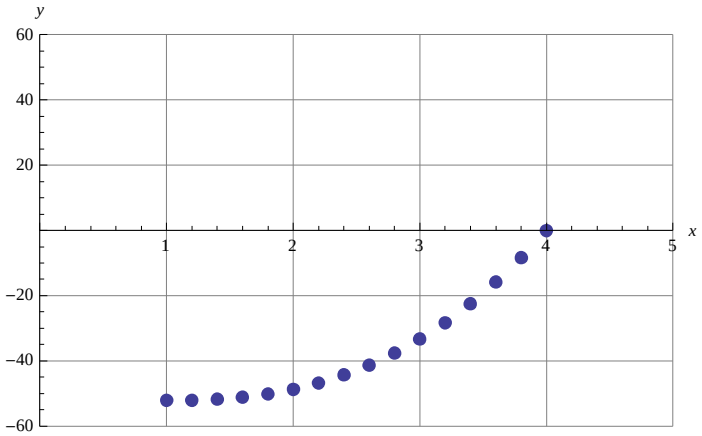
```
vecy = Inverse[listmatAb].listvecCb
```

$$\left\{-\frac{1302}{25}, -\frac{1302}{25}, -\frac{6474}{125}, -\frac{6396}{125}, -\frac{1254}{25}, -\frac{1218}{25}, -\frac{234}{5}, -\frac{5544}{125}, -\frac{5166}{125}, -\frac{942}{25}, -\frac{834}{25}, -\frac{708}{25}, -\frac{2814}{125}, -\frac{1986}{125}, -\frac{42}{5}, 0\right\}$$

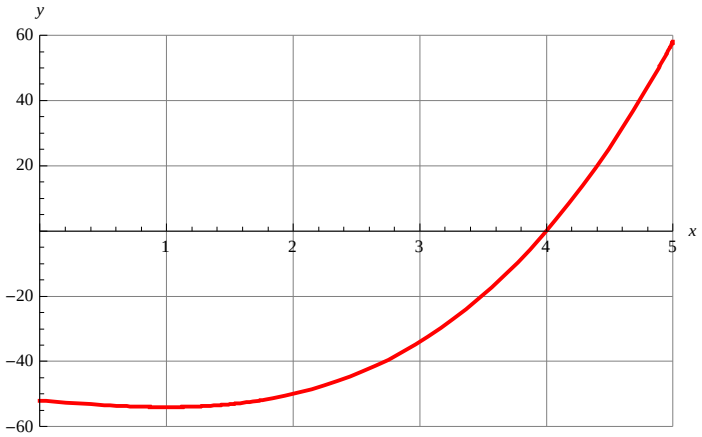
```
data = Table[{xn[[i]], vecy[[i]]}, {i, 1, nx}]
```

$$\left\{\left\{1, -\frac{1302}{25}\right\}, \left\{\frac{6}{5}, -\frac{1302}{25}\right\}, \left\{\frac{7}{5}, -\frac{6474}{125}\right\}, \left\{\frac{8}{5}, -\frac{6396}{125}\right\}, \left\{\frac{9}{5}, -\frac{1254}{25}\right\},\right.$$
$$\left\{2, -\frac{1218}{25}\right\}, \left\{\frac{11}{5}, -\frac{234}{5}\right\}, \left\{\frac{12}{5}, -\frac{5544}{125}\right\}, \left\{\frac{13}{5}, -\frac{5166}{125}\right\}, \left\{\frac{14}{5}, -\frac{942}{25}\right\},$$
$$\left.\left\{3, -\frac{834}{25}\right\}, \left\{\frac{16}{5}, -\frac{708}{25}\right\}, \left\{\frac{17}{5}, -\frac{2814}{125}\right\}, \left\{\frac{18}{5}, -\frac{1986}{125}\right\}, \left\{\frac{19}{5}, -\frac{42}{5}\right\}, \{4, 0\}\right\}$$

```
g0 = ListPlot[data, GridLines -> Automatic, AxesLabel → {x, y},
  PlotStyle → PointSize[Large], PlotRange → {{0, 5}, {-60, 60}}]
```




```
g1 = Plot[x^3 - 3 x - 52, {x, 0, 5}, AxesLabel → {x, y},
  PlotStyle → {Red, Thick}, GridLines -> Automatic, PlotRange → {{0, 5}, {-60, 60}}]
```




```
Show[g1, g0]
```