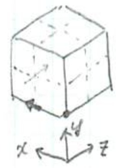


$$\boxed{\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x\right)}$$

$n-\frac{1}{2}$, $\boxed{i+\frac{1}{2}, j, k}$

$$\frac{E_x^n(i+\frac{1}{2},j,k) - E_x^{n-1}(i+\frac{1}{2},j,k)}{\Delta t} = \underbrace{\frac{1}{\varepsilon(i+\frac{1}{2},j,k)}\left\{\frac{H_z^{n-\frac{1}{2}}(i+\frac{1}{2},j+\frac{1}{2},k) - H_z^{n-\frac{1}{2}}(i+\frac{1}{2},j-\frac{1}{2},k)}{\Delta y} - \frac{H_y^{n-\frac{1}{2}}(i+\frac{1}{2},j,k+\frac{1}{2}) - H_y^{n-\frac{1}{2}}(i+\frac{1}{2},j,k-\frac{1}{2})}{\Delta z}\right\}}_{ⓐ}$$

$$\underbrace{- \frac{1}{\varepsilon_x(i+\frac{1}{2},j,k)}\sigma_x(i+\frac{1}{2},j,k)\, E_x^{n-\frac{1}{2}}(i+\frac{1}{2},j,k)}_{ⓑ}$$

$\sigma E^{n-\frac{1}{2}} \Rightarrow \sigma \dfrac{E^{n-1}+E^n}{2}$ を使うと，

$$ⓑ \Rightarrow -\frac{\sigma_x(i+\frac{1}{2},j,k)}{\varepsilon_x(i+\frac{1}{2},j,k)}\,\frac{E_x^{n-1}(i+\frac{1}{2},j,k) + E_x^n(i+\frac{1}{2},j,k)}{2}$$

空間差分は式を整理し終ってから．

Isotropic media

$$\varepsilon = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix},\quad \varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon.$$

$$\sigma = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix},\quad \sigma_x = \sigma_y = \sigma_z = \sigma$$

$n-\frac{1}{2}$

時間微分だけ先に行うと，

$$\frac{E_x^n - E_x^{n-1}}{\Delta t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z^{n-\frac{1}{2}}}{\partial y} - \frac{\partial H_y^{n-\frac{1}{2}}}{\partial z} - \sigma E_x^{n-\frac{1}{2}}\right)$$

$$= \frac{1}{\varepsilon}\left(\frac{\partial H_z^{n-\frac{1}{2}}}{\partial y} - \frac{\partial H_y^{n-\frac{1}{2}}}{\partial z} - \frac{\sigma E_x^{n-1} + \sigma E_x^n}{2}\right)$$

$$= \frac{1}{\varepsilon}\left(\frac{\partial H_z^{n-\frac{1}{2}}}{\partial y} - \frac{\partial H_y^{n-\frac{1}{2}}}{\partial z}\right) - \frac{\sigma}{2\varepsilon}E_x^{n-1} - \frac{\sigma}{2\varepsilon}E_x^n$$

$\Rightarrow E_x^n$ について整理すると，

$$E_x^n - E_x^{n-1} = \frac{\Delta t}{\varepsilon}\left(\frac{\partial H_z^{n-\frac{1}{2}}}{\partial y} - \frac{\partial H_y^{n-\frac{1}{2}}}{\partial z}\right) - \frac{\Delta t\,\sigma}{2\varepsilon}E_x^{n-1} - \frac{\Delta t\,\sigma}{2\varepsilon}E_x^n$$

$$\Rightarrow \left(1 + \frac{\sigma\Delta t}{2\varepsilon}\right)E_x^n = \frac{\Delta t}{\varepsilon}\left(\frac{\partial H_z^{n-\frac{1}{2}}}{\partial y} - \frac{\partial H_y^{n-\frac{1}{2}}}{\partial z}\right) + \left(1 - \frac{\sigma\Delta t}{2\varepsilon}\right)E_x^{n-1}$$

$$\boxed{E_x^n = \frac{\frac{\Delta t}{\varepsilon}}{1 + \frac{\sigma\Delta t}{2\varepsilon}}\left(\frac{\partial H_z^{n-\frac{1}{2}}}{\partial y} - \frac{\partial H_y^{n-\frac{1}{2}}}{\partial z}\right) + \frac{1 - \frac{\sigma\Delta t}{2\varepsilon}}{1 + \frac{\sigma\Delta t}{2\varepsilon}}E_x^{n-1}} \qquad (1.44)$$

宇野

$\boxed{i+\frac{1}{2}, j, k}$

空間微分を別々に行うと，

$$\begin{cases} \dfrac{\partial H_z^{n-\frac{1}{2}}}{\partial y} = \dfrac{1}{\Delta y}\left\{H_z^{n-\frac{1}{2}}(i+\frac{1}{2},j+\frac{1}{2},k) - H_z^{n-\frac{1}{2}}(i+\frac{1}{2},j-\frac{1}{2},k)\right\} \\[2ex] \dfrac{\partial H_y^{n-\frac{1}{2}}}{\partial z} = \dfrac{1}{\Delta z}\left\{H_y^{n-\frac{1}{2}}(i+\frac{1}{2},j,k+\frac{1}{2}) - H_y^{n-\frac{1}{2}}(i+\frac{1}{2},j,k-\frac{1}{2})\right\} \\[2ex] E_x^n = E_x^n(i+\frac{1}{2},j,k) \\[1ex] E_x^{n-1} = E_x^{n-1}(i+\frac{1}{2},j,k) \end{cases}$$

係数

$$\frac{1 - \dfrac{\sigma(i+\frac{1}{2},j,k)\Delta t}{2\varepsilon(i+\frac{1}{2},j,k)}}{1 + \dfrac{\sigma(i+\frac{1}{2},j,k)\Delta t}{2\varepsilon(i+\frac{1}{2},j,k)}} = cex$$

$$\frac{\dfrac{\Delta t}{\varepsilon(i+\frac{1}{2},j,k)}}{1 + \dfrac{\sigma(i+\frac{1}{2},j,k)\Delta t}{2\varepsilon(i+\frac{1}{2},j,k)}}\cdot\frac{1}{\Delta y} = cexly$$

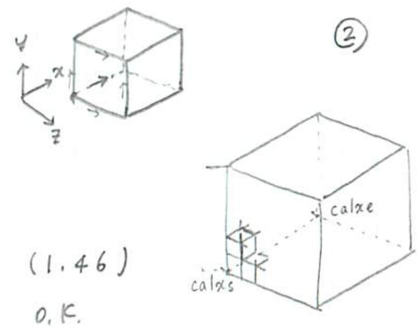
$$\frac{\dfrac{\Delta t}{\varepsilon(i+\frac{1}{2},j,k)}}{1 + \dfrac{\sigma(i+\frac{1}{2},j,k)\Delta t}{2\varepsilon(i+\frac{1}{2},j,k)}}\cdot\frac{1}{\Delta z} = cexlz$$

②

最終的に,

$$
\begin{aligned}
E_x^{n}(\overset{I}{i+\tfrac12},\overset{J}{j},\overset{K}{k}) = {} & cex\cdot E_x^{n-1}(\overset{I}{i+\tfrac12},\overset{J}{j},\overset{K}{k}) \\
& + cexly\cdot\left\{H_z^{n-\frac12}(\overset{I}{i+\tfrac12},\overset{J}{j+\tfrac12},\overset{K}{k}) - H_z^{n-\frac12}(\overset{I}{i+\tfrac12},\overset{J-1}{j-\tfrac12},\overset{K}{k})\right\} \\
& - cexlz\cdot\left\{H_y^{n-\frac12}(\overset{I}{i+\tfrac12},\overset{J}{j},\overset{K}{k+\tfrac12}) - H_y^{n-\frac12}(\overset{I}{i+\tfrac12},\overset{J}{j},\overset{K-1}{k-\tfrac12})\right\}
\end{aligned}
\qquad (1.46)
$$

o.K.

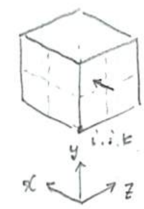


x; calxs, calxe-1.
y; calys, calye
z; calzs, calze.

$$\boxed{\frac{\partial H_x}{\partial t} = -\frac{1}{\mu}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)}$$

nで時間差分をとると,

$$\frac{H_x^{n+\frac12} - H_x^{n-\frac12}}{\Delta t} = -\frac{1}{\mu}\left(\frac{\partial E_z^n}{\partial y} - \frac{\partial E_y^n}{\partial z}\right)$$



$\Rightarrow$ $H_x^{n+\frac12}$ について整理すると,

$$H_x^{n+\frac12} - H_x^{n-\frac12} = -\frac{\Delta t}{\mu}\left(\frac{\partial E_z^n}{\partial y} - \frac{\partial E_y^n}{\partial z}\right)$$

$$\boxed{H_x^{n+\frac12} = H_x^{n-\frac12} - \frac{\Delta t}{\mu}\left(\frac{\partial E_z^n}{\partial y} - \frac{\partial E_y^n}{\partial z}\right)} \qquad (1.48)$$

$\boxed{i, j+\frac12, k+\frac12}$ で空間差分をとると,

$$
\left\{
\begin{aligned}
\frac{\partial E_z^n}{\partial y} &= \frac{1}{\Delta y}\left\{E_z^n(i, j+1, k+\tfrac12) - E_z^n(i, j, k+\tfrac12)\right\} \\
\frac{\partial E_y^n}{\partial z} &= \frac{1}{\Delta z}\left\{E_y^n(i, j+\tfrac12, k+1) - E_y^n(i, j+\tfrac12, k)\right\} \\
H_x^{n+\frac12} &= H_x^{n+\frac12}(i, j+\tfrac12, k+\tfrac12) \\
H_x^{n-\frac12} &= H_x^{n-\frac12}(i, j+\tfrac12, k+\tfrac12)
\end{aligned}
\right.
$$

係数

$$\frac{\Delta t}{\mu(i,j+\frac12,k+\frac12)}\cdot\frac{1}{\Delta y} = chxly$$

$$\frac{\Delta t}{\mu(i,j+\frac12,k+\frac12)}\cdot\frac{1}{\Delta z} = chxlz \qquad (1.55a)$$

最終的に,

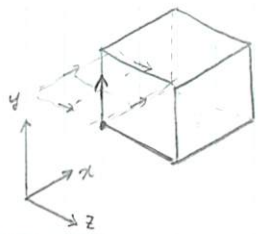
$$
\begin{aligned}
H_x^{n+\frac12}(\overset{I}{i},\overset{J}{j+\tfrac12},\overset{K}{k+\tfrac12}) = {} & H_x^{n-\frac12}(\overset{I}{i},\overset{J}{j+\tfrac12},\overset{K}{k+\tfrac12}) \\
& - chxly\left\{E_z^n(\overset{I}{i},\overset{J+1}{j+1},\overset{K}{k+\tfrac12}) - E_z^n(\overset{I}{i},\overset{J}{j},\overset{K}{k+\tfrac12})\right\} \\
& + chxlz\left\{E_y^n(\overset{I}{i},\overset{J}{j+\tfrac12},\overset{K+1}{k+1}) - E_y^n(\overset{I}{i},\overset{J}{j+\tfrac12},\overset{K}{k})\right\}
\end{aligned}
\qquad (1.50) \quad (1.54a)
$$

o.K.

x; calxs, calxe
y; calys, calye-1.
z; calzs, calze-1.

$$\boxed{\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y\right)}$$

$n-\frac{1}{2}$ で時間微分を先に行う.



$$\boxed{E_y^n = \frac{\frac{\Delta t}{\varepsilon}}{1+\frac{\sigma\Delta t}{2\varepsilon}}\left(\frac{\partial H_x^{n-\frac{1}{2}}}{\partial z} - \frac{\partial H_z^{n-\frac{1}{2}}}{\partial x}\right) + \frac{1-\frac{\sigma\Delta t}{2\varepsilon}}{1+\frac{\sigma\Delta t}{2\varepsilon}} E_y^{n-1}} \qquad (1.44)$$

$\boxed{i, j+\frac{1}{2}, k}$ で空間微分

$$\left\{\begin{aligned}
\frac{\partial H_x^{n-\frac{1}{2}}}{\partial z} &= \frac{1}{\Delta z}\left\{H_x^{n-\frac{1}{2}}(i, j+\tfrac{1}{2}, k+\tfrac{1}{2}) - H_x^{n-\frac{1}{2}}(i, j+\tfrac{1}{2}, k-\tfrac{1}{2})\right\} \\
\frac{\partial H_z^{n-\frac{1}{2}}}{\partial x} &= \frac{1}{\Delta x}\left\{H_z^{n-\frac{1}{2}}(i+\tfrac{1}{2}, j+\tfrac{1}{2}, k) - H_z^{n-\frac{1}{2}}(i-\tfrac{1}{2}, j+\tfrac{1}{2}, k)\right\} \\
E_y^n &= E_y^n(i, j+\tfrac{1}{2}, k) \\
E_y^{n-1} &= E_y^n(i, j+\tfrac{1}{2}, k)
\end{aligned}\right.$$

係数

$$cey = \frac{1-\frac{\sigma(\overset{I}{i}, \overset{J}{j+\frac{1}{2}}, \overset{K}{k})\Delta t}{2\varepsilon(i, j+\frac{1}{2}, k)}}{1+\frac{\sigma(i, j+\frac{1}{2}, k)\Delta t}{2\varepsilon(i, j+\frac{1}{2}, k)}} \qquad (1.53b)$$

$$ceylz = \frac{\frac{\Delta t}{\varepsilon(i, j+\frac{1}{2}, k)}}{1+\frac{\sigma(i, j+\frac{1}{2}, k)\Delta t}{2\varepsilon(i, j+\frac{1}{2}, k)}}\cdot\frac{1}{\Delta z}, \qquad ceylx = \frac{\frac{\Delta t}{\varepsilon(i, j+\frac{1}{2}, k)}}{1+\frac{\sigma(i, j+\frac{1}{2}, k)\Delta t}{2\varepsilon(i, j+\frac{1}{2}, k)}}\cdot\frac{1}{\Delta x} \qquad (1.53b)$$

O.K.

最終的に.

$$\begin{aligned}
E_y^n(\overset{I}{i}, \overset{J}{j+\tfrac{1}{2}}, \overset{K}{k}) = {}& cey\cdot E_y^{n-1}(\overset{I}{i}, \overset{J}{j+\tfrac{1}{2}}, \overset{K}{k}) \\
&+ ceylz\cdot\left\{H_x^{n-\frac{1}{2}}(\overset{I}{i}, \overset{J}{j+\tfrac{1}{2}}, \overset{K}{k+\tfrac{1}{2}}) - H_x^{n-\frac{1}{2}}(\overset{I}{i}, \overset{J}{j+\tfrac{1}{2}}, \overset{K-1}{k-\tfrac{1}{2}})\right\} \\
&- ceylx\left\{H_x^{n-\frac{1}{2}}(\overset{I}{i+\tfrac{1}{2}}, \overset{J}{j+\tfrac{1}{2}}, \overset{K}{k}) - H_z^{n-\frac{1}{2}}(\overset{I-1}{i-\tfrac{1}{2}}, \overset{J}{j+\tfrac{1}{2}}, \overset{K}{k})\right\}
\end{aligned} \qquad (1.52b)$$

O.K.

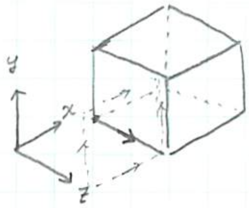
O.K.

x: calxs, calxe

y: calys, calye-1.

z: calzs, calze

$$\boxed{\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z\right)}$$



$n-\frac{1}{2}$ で時間微分.

$$\boxed{E_z^{n} = \frac{\frac{\Delta t}{\varepsilon}}{1+\frac{\sigma\Delta t}{2\varepsilon}}\left(\frac{\partial H_y^{n-\frac{1}{2}}}{\partial x} - \frac{\partial H_x^{n-\frac{1}{2}}}{\partial y}\right) + \frac{1-\frac{\sigma\Delta t}{2\varepsilon}}{1+\frac{\sigma\Delta t}{2\varepsilon}} E_z^{n-1}} \qquad (1.44)$$

$\boxed{i, j, k+\frac{1}{2}}$ で空間微分.

$$\begin{cases} \dfrac{\partial H_y^{n-\frac{1}{2}}}{\partial x} = \dfrac{1}{\Delta x}\left\{ H_y^{n-\frac{1}{2}}(i+\frac{1}{2}, j, k+\frac{1}{2}) - H_y^{n-\frac{1}{2}}(i-\frac{1}{2}, j, k+\frac{1}{2}) \right\} \\[2ex] \dfrac{\partial H_x^{n-\frac{1}{2}}}{\partial y} = \dfrac{1}{\Delta y}\left\{ H_x^{n-\frac{1}{2}}(i, j+\frac{1}{2}, k+\frac{1}{2}) - H_x^{n-\frac{1}{2}}(i, j-\frac{1}{2}, k+\frac{1}{2}) \right\} \\[2ex] E_z^{n} = E_z^{n}(i, j, k+\frac{1}{2}) \\[2ex] E_z^{n-1} = E_z^{n-1}(i, j, k+\frac{1}{2}) \end{cases}$$

- 係数

$$cez = \frac{1-\dfrac{\sigma(\overset{I}{i},\overset{J}{j},\overset{K}{k+\frac{1}{2}})\Delta t}{2\varepsilon(i,j,k+\frac{1}{2})}}{1+\dfrac{\sigma(i,j,k+\frac{1}{2})\Delta t}{2\varepsilon(i,j,k+\frac{1}{2})}} \qquad (1.47a)$$

$$cezlx = \frac{\dfrac{\Delta t}{\varepsilon(i,j,k+\frac{1}{2})}}{1+\dfrac{\sigma(i,j,k+\frac{1}{2})\Delta t}{2\varepsilon(i,j,k+\frac{1}{2})}}\cdot\frac{1}{\Delta x}, \qquad cezly = \frac{\dfrac{\Delta t}{\varepsilon(i,j,k+\frac{1}{2})}}{1+\dfrac{\sigma(i,j,k+\frac{1}{2})\Delta t}{2\varepsilon(i,j,k+\frac{1}{2})}}\cdot\frac{1}{\Delta y} \qquad (1.47b)$$

o.k.

- Ez の差分式

$$\begin{aligned} E_z^{n}(\overset{I}{i},\overset{J}{j},\overset{K}{k+\tfrac{1}{2}}) = {} & cez\cdot E_z^{n-1}(\overset{I}{i},\overset{J}{j},\overset{K}{k+\tfrac{1}{2}}) \\ & + cezlx\cdot\left\{ H_y^{n-\frac{1}{2}}(\overset{I}{i+\tfrac{1}{2}},\overset{J}{j},\overset{K}{k+\tfrac{1}{2}}) - H_y^{n-\frac{1}{2}}(\overset{I-1}{i-\tfrac{1}{2}},\overset{J}{j},\overset{K}{k+\tfrac{1}{2}}) \right\} \\ & - cezly\cdot\left\{ H_x^{n-\frac{1}{2}}(\overset{I}{i},\overset{J}{j+\tfrac{1}{2}},\overset{K}{k+\tfrac{1}{2}}) - H_x^{n-\frac{1}{2}}(\overset{I}{i},\overset{J-1}{j-\tfrac{1}{2}},\overset{K}{k+\tfrac{1}{2}}) \right\} \end{aligned} \qquad (1.46)$$

o.k.

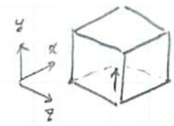
o.k.

x : calxs, calxe.

y : calys, calye.

z : calzs, calze - 1.

$$\boxed{\frac{\partial H_y}{\partial t} = -\frac{1}{\mu}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)}$$



⑭で時間微分を先に行う.

$$H_y^{n+\frac{1}{2}} = H_y^{n-\frac{1}{2}} - \frac{\Delta t}{\mu}\left(\frac{\partial E_x^n}{\partial z} - \frac{\partial E_z^n}{\partial x}\right) \qquad (1.48)$$

$\boxed{i+\frac{1}{2}, j, k+\frac{1}{2}}$ で空間微分をとる.

$$\begin{cases} \dfrac{\partial E_x^n}{\partial z} = \dfrac{1}{\Delta z}\left\{ E_x^n(i+\frac{1}{2}, j, k+1) - E_x^n(i+\frac{1}{2}, j, k) \right\} \\ \dfrac{\partial E_z^n}{\partial x} = \dfrac{1}{\Delta x}\left\{ E_z^n(i+1, j, k+\frac{1}{2}) - E_z^n(i, j, k+\frac{1}{2}) \right\} \\ H_y^{n+\frac{1}{2}} = H_y^{n+\frac{1}{2}}(i+\frac{1}{2}, j, k+\frac{1}{2}) \\ H_y^{n-\frac{1}{2}} = H_y^{n-\frac{1}{2}}(i+\frac{1}{2}, j, k+\frac{1}{2}) \end{cases}$$

係数.

$$chylz = \frac{\Delta t}{\mu(i+\frac{1}{2}, j, k+\frac{1}{2})}\cdot\frac{1}{\Delta z}, \quad chylx = \frac{\Delta t}{\mu(i+\frac{1}{2}, j, k+\frac{1}{2})}\cdot\frac{1}{\Delta x} \qquad (1.55b)$$

o.K.

最終的に.

$$\begin{aligned} H_y^{n+\frac{1}{2}}(\overset{I}{i+\tfrac{1}{2}}, \overset{J}{j}, \overset{K}{k+\tfrac{1}{2}}) = {} & H_y^{n-\frac{1}{2}}(\overset{I}{i+\tfrac{1}{2}}, \overset{J}{j}, \overset{K}{k+\tfrac{1}{2}}) \\ & - chylz\left\{ E_x^n(\overset{I}{i+\tfrac{1}{2}}, \overset{J}{j}, \overset{K+1}{k+1}) - E_x^n(\overset{I}{i+\tfrac{1}{2}}, \overset{J}{j}, \overset{K}{k}) \right\} \\ & + chylx\left\{ E_z^n(\overset{I+1}{i+1}, \overset{J}{j}, \overset{K}{k+\tfrac{1}{2}}) - E_z^n(\overset{I}{i}, \overset{J}{j}, \overset{K}{k+\tfrac{1}{2}}) \right\} \end{aligned} \qquad (1.54b)$$

o.K.

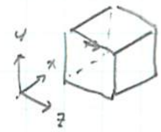
o.K.

x : calxs, calxe-1.

y : calys, calye

z : calzs, calze-1.

$$\boxed{\frac{\partial H_z}{\partial t} = -\frac{1}{\mu}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)}$$



ⓝ で時間微分を先に行うと.

$$H_z^{n+\frac{1}{2}} = H_z^{n-\frac{1}{2}} - \frac{\Delta t}{\mu}\left(\frac{\partial E_y^n}{\partial x} - \frac{\partial E_x^n}{\partial y}\right) \qquad (1.48)$$

$\boxed{i+\frac{1}{2},\ j+\frac{1}{2},\ k}$ で空間微分を行う.

$$\begin{cases}
\dfrac{\partial E_y^n}{\partial x} = \dfrac{1}{\Delta x}\left\{E_y^n\left(i+1, j+\frac{1}{2}, k\right) - E_y^n\left(i, j+\frac{1}{2}, k\right)\right\} \\[2ex]
\dfrac{\partial E_x^n}{\partial y} = \dfrac{1}{\Delta y}\left\{E_x^n\left(i+\frac{1}{2}, j+1, k\right) - E_x^n\left(i+\frac{1}{2}, j, k\right)\right\} \\[2ex]
H_z^{n+\frac{1}{2}} = H_z^{n+\frac{1}{2}}\left(i+\frac{1}{2}, j+\frac{1}{2}, k\right) \\[2ex]
H_z^{n-\frac{1}{2}} = H_z^{n-\frac{1}{2}}\left(i+\frac{1}{2}, j+\frac{1}{2}, k\right)
\end{cases}$$

係数

$$chzlx = \frac{\Delta t}{\mu\left(i+\frac{1}{2}, j+\frac{1}{2}, k\right)} \cdot \frac{1}{\Delta x}, \qquad chzly = \frac{\Delta t}{\mu\left(i+\frac{1}{2}, j+\frac{1}{2}, k\right)} \cdot \frac{1}{\Delta y} \qquad (1.55c)$$

O.K.

最終的に.

$$\begin{aligned}
H_z^{n+\frac{1}{2}}\left(\overset{I}{i+\tfrac{1}{2}}, \overset{J}{j+\tfrac{1}{2}}, \overset{K}{k}\right) &= H_z^{n-\frac{1}{2}}\left(\overset{I}{i+\tfrac{1}{2}}, \overset{J}{j+\tfrac{1}{2}}, \overset{K}{k}\right) \\
&- chzlx\left\{E_y^n\left(\overset{I+1}{i+1}, \overset{J}{j+\tfrac{1}{2}}, \overset{K}{k}\right) - E_y^n\left(\overset{I}{i}, \overset{J}{j+\tfrac{1}{2}}, \overset{K}{k}\right)\right\} \\
&+ chzly\left\{E_x^n\left(\overset{I}{i+\tfrac{1}{2}}, \overset{J+1}{j+1}, \overset{K}{k}\right) - E_y^n\left(\overset{I}{i+\tfrac{1}{2}}, \overset{J}{j}, \overset{K}{k}\right)\right\}
\end{aligned} \qquad (1.54c)$$

O.K.

O.K.

x : calxs, calxe-1.

y : calys, calye-1

z : calzs, calze

$$\boxed{\frac{\partial H_x}{\partial t} = -\frac{1}{\mu}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} + \sigma^* H_x\right)}$$

ⓝ で時間微分すると、

$$\boxed{H_x^{n+\frac{1}{2}} = -\frac{\frac{\Delta t}{\mu}}{1+\frac{\sigma^*\Delta t}{2\mu}}\left(\frac{\partial E_z^n}{\partial y} - \frac{\partial E_y^n}{\partial z}\right) + \frac{1-\frac{\sigma^*\Delta t}{2\mu}}{1+\frac{\sigma^*\Delta t}{2\mu}} H_x^{n-\frac{1}{2}}}$$

$i, j+\frac{1}{2}, k+\frac{1}{2}$ で空間微分すると、

$$\left\{\begin{aligned}
\frac{\partial E_z^n}{\partial y} &= \frac{1}{\Delta y}\left\{E_z^n\left(i, j+1, k+\tfrac{1}{2}\right) - E_z^n\left(i, j, k+\tfrac{1}{2}\right)\right\} \\
\frac{\partial E_y^n}{\partial z} &= \frac{1}{\Delta z}\left\{E_y^n\left(i, j+\tfrac{1}{2}, k+1\right) - E_y^n\left(i, j+\tfrac{1}{2}, k\right)\right\} \\
H_x^{n+\frac{1}{2}} &= H_x^{n+\frac{1}{2}}\left(i, j+\tfrac{1}{2}, k+\tfrac{1}{2}\right) \\
H_x^{n-\frac{1}{2}} &= H_x^{n-\frac{1}{2}}\left(i, j+\tfrac{1}{2}, k+\tfrac{1}{2}\right)
\end{aligned}\right.$$

係数

$$\frac{1-\frac{\sigma^*(i, j+\frac{1}{2}, k+\frac{1}{2})\Delta t}{2\mu(i, j+\frac{1}{2}, k+\frac{1}{2})}}{1+\frac{\sigma^*(i, j+\frac{1}{2}, k+\frac{1}{2})\Delta t}{2\mu(i, j+\frac{1}{2}, k+\frac{1}{2})}} = chx$$

$$\frac{\frac{\Delta t}{\mu(i, j+\frac{1}{2}, k+\frac{1}{2})}}{1+\frac{\sigma^*(i, j+\frac{1}{2}, k+\frac{1}{2})\Delta t}{2\mu(i, j+\frac{1}{2}, k+\frac{1}{2})}} \cdot \frac{1}{\Delta y} = chxly$$

$$\frac{\frac{\Delta t}{\mu(i, j+\frac{1}{2}, k+\frac{1}{2})}}{1+\frac{\sigma^*(i, j+\frac{1}{2}, k+\frac{1}{2})\Delta t}{2\mu(i, j+\frac{1}{2}, k+\frac{1}{2})}} \cdot \frac{1}{\Delta z} = chxlz$$

最終的に

$$\begin{aligned}
H_x^{n+\frac{1}{2}}\left(i, j+\tfrac{1}{2}, k+\tfrac{1}{2}\right) = {} & chx \cdot H_x^{n-\frac{1}{2}}\left(i, j+\tfrac{1}{2}, k+\tfrac{1}{2}\right) \\
& - chxly\left\{E_z^n\left(i, j+1, k+\tfrac{1}{2}\right) - E_z^n\left(i, j, k+\tfrac{1}{2}\right)\right\} \\
& + chxlz\left\{E_y^n\left(i, j+\tfrac{1}{2}, k+1\right) - E_y^n\left(i, j+\tfrac{1}{2}, k\right)\right\}
\end{aligned}$$

$$\boxed{\frac{\partial H_y}{\partial t} = -\frac{1}{\mu}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} + \sigma^* H_y\right)}$$

ⓝで時間微分をとると、

$$\frac{H_y^{n+\frac{1}{2}} - H_y^{n-\frac{1}{2}}}{\Delta t} = -\frac{1}{\mu}\left(\frac{\partial E_x^n}{\partial z} - \frac{\partial E_z^n}{\partial x} + \sigma^* H_y^n\right)$$

$$= -\frac{1}{\mu}\left(\frac{\partial E_x^n}{\partial z} - \frac{\partial E_z^n}{\partial x} + \frac{\sigma^* H_y^{n+\frac{1}{2}} + \sigma^* H_y^{n-\frac{1}{2}}}{2}\right)$$

$$= -\frac{1}{\mu}\left(\frac{\partial E_x^n}{\partial z} - \frac{\partial E_z^n}{\partial x}\right) - \frac{\sigma^*}{2\mu} H_y^{n+\frac{1}{2}} - \frac{\sigma^*}{2\mu} H_y^{n-\frac{1}{2}}$$

⇒ $H_y^{n+\frac{1}{2}}$ について整理すると、

$$H_y^{n+\frac{1}{2}} - H_y^{n-\frac{1}{2}} = -\frac{\Delta t}{\mu}\left(\frac{\partial E_x^n}{\partial z} - \frac{\partial E_z^n}{\partial x}\right) - \frac{\Delta t \sigma^*}{2\mu} H_y^{n+\frac{1}{2}} - \frac{\Delta t \sigma^*}{2\mu} H_y^{n-\frac{1}{2}}$$

$$\left(1 + \frac{\Delta t \sigma^*}{2\mu}\right) H_y^{n+\frac{1}{2}} = -\frac{\Delta t}{\mu}\left(\frac{\partial E_x^n}{\partial z} - \frac{\partial E_z^n}{\partial x}\right) + \left(1 - \frac{\Delta t \sigma^*}{2\mu}\right) H_y^{n-\frac{1}{2}}$$

$$\boxed{H_y^{n+\frac{1}{2}} = -\frac{\frac{\Delta t}{\mu}}{1 + \frac{\Delta t \sigma^*}{2\mu}}\left(\frac{\partial E_x^n}{\partial z} - \frac{\partial E_z^n}{\partial x}\right) + \frac{1 - \frac{\sigma^* \Delta t}{2\mu}}{1 + \frac{\sigma^* \Delta t}{2\mu}} H_y^{n-\frac{1}{2}}}$$

係数

$i+\frac{1}{2}, j, k+\frac{1}{2}$ で空間微分をとると、

$$\begin{cases} \dfrac{\partial E_x^n}{\partial z} = \dfrac{1}{\Delta z}\left\{E_x^n(i+\frac{1}{2}, j, k+1) - E_x^n(i+\frac{1}{2}, j, k)\right\} \\ \dfrac{\partial E_z^n}{\partial x} = \dfrac{1}{\Delta x}\left\{E_z^n(i+1, j, k+\frac{1}{2}) - E_z^n(i, j, k+\frac{1}{2})\right\} \\ H_y^{n+\frac{1}{2}} = H_y^{n+\frac{1}{2}}(i+\frac{1}{2}, j, k+\frac{1}{2}) \\ H_y^{n-\frac{1}{2}} = H_y^{n-\frac{1}{2}}(i+\frac{1}{2}, j, k+\frac{1}{2}) \end{cases}$$

$$\frac{1 - \dfrac{\sigma^*(i+\frac{1}{2}, j, k+\frac{1}{2})\Delta t}{2\mu(i+\frac{1}{2}, j, k+\frac{1}{2})}}{1 + \dfrac{\sigma^*(i+\frac{1}{2}, j, k+\frac{1}{2})\Delta t}{2\mu(i+\frac{1}{2}, j, k+\frac{1}{2})}} = chy$$

$$\frac{\dfrac{\Delta t}{\mu(i+\frac{1}{2}, j, k+\frac{1}{2})}}{1 + \dfrac{\sigma^*(i+\frac{1}{2}, j, k+\frac{1}{2})\Delta t}{2\mu(i+\frac{1}{2}, j, k+\frac{1}{2})}} \cdot \frac{1}{\Delta z} = chylz$$

最終的に、

$$H_y^{n+\frac{1}{2}}(i+\frac{1}{2}, j, k+\frac{1}{2}) = chy \cdot H_y^{n-\frac{1}{2}}(i+\frac{1}{2}, j, k+\frac{1}{2})$$
$$- chylz\left\{E_x^n(i+\frac{1}{2}, j, k+1) - E_x^n(i+\frac{1}{2}, j, k)\right\}$$
$$+ chylx\left\{E_z^n(i+1, j, k+\frac{1}{2}) - E_z^n(i, j, k+\frac{1}{2})\right\}$$

$$\frac{\dfrac{\Delta t}{\mu(i+\frac{1}{2}, j, k+\frac{1}{2})}}{1 + \dfrac{\sigma^*(i+\frac{1}{2}, j, k+\frac{1}{2})\Delta t}{2\mu(i+\frac{1}{2}, j, k+\frac{1}{2})}} \cdot \frac{1}{\Delta x} = chylx$$

$$\boxed{\frac{\partial H_z}{\partial t} = -\frac{1}{\mu}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \sigma^* H_z\right)}$$

ⓝ で時間微分すると、

$$\boxed{H_z^{n+\frac{1}{2}} = -\frac{\frac{\Delta t}{\mu}}{1+\frac{\sigma^*\Delta t}{2\mu}}\left(\frac{\partial E_y^n}{\partial x} - \frac{\partial E_x^n}{\partial y}\right) + \frac{1-\frac{\sigma^*\Delta t}{2\mu}}{1+\frac{\sigma^*\Delta t}{2\mu}} H_z^{n-\frac{1}{2}}}$$

係数

$i+\frac{1}{2}, j+\frac{1}{2}, k$ で空間微分すると、

$$\left\{\begin{aligned}
\frac{\partial E_y^n}{\partial x} &= \frac{1}{\Delta x}\left\{E_y^n\left(i+1, j+\tfrac{1}{2}, k\right) - E_y^n\left(i, j+\tfrac{1}{2}, k\right)\right\} \\
\frac{\partial E_x^n}{\partial y} &= \frac{1}{\Delta y}\left\{E_x^n\left(i+\tfrac{1}{2}, j+1, k\right) - E_x^n\left(i+\tfrac{1}{2}, j+1, k\right)\right\} \\
H_z^{n+\frac{1}{2}} &= H_z^{n+\frac{1}{2}}\left(i+\tfrac{1}{2}, j+\tfrac{1}{2}, k\right) \\
H_z^{n-\frac{1}{2}} &= H_z^{n-\frac{1}{2}}\left(i+\tfrac{1}{2}, j+\tfrac{1}{2}, k\right)
\end{aligned}\right.$$

$$\frac{1-\frac{\sigma^*(i+\frac{1}{2}, j+\frac{1}{2}, k)\Delta t}{2\mu(i+\frac{1}{2}, j+\frac{1}{2}, k)}}{1+\frac{\sigma^*(i+\frac{1}{2}, j+\frac{1}{2}, k)\Delta t}{2\mu(i+\frac{1}{2}, j+\frac{1}{2}, k)}} = chz$$

$$\frac{\frac{\Delta t}{\mu(i+\frac{1}{2}, j+\frac{1}{2}, k)}}{1+\frac{\sigma^*(i+\frac{1}{2}, j+\frac{1}{2}, k)\Delta t}{2\mu(i+\frac{1}{2}, j+\frac{1}{2}, k)}} \cdot \frac{1}{\Delta x} = chzlx$$

$$\frac{\frac{\Delta t}{\mu(i+\frac{1}{2}, j+\frac{1}{2}, k)}}{1+\frac{\sigma^*(i+\frac{1}{2}, j+\frac{1}{2}, k)\Delta t}{2\mu(i+\frac{1}{2}, j+\frac{1}{2}, k)}} \cdot \frac{1}{\Delta y} = chzly$$

最終的に、

$$\begin{aligned}
H_z^{n+\frac{1}{2}}\left(i+\tfrac{1}{2}, j+\tfrac{1}{2}, k\right) &= chz \cdot H_z^{n-\frac{1}{2}}\left(i+\tfrac{1}{2}, j+\tfrac{1}{2}, k\right) \\
&\quad - chzlx\left\{E_y^n\left(i+1, j+\tfrac{1}{2}, k\right) - E_y^n\left(i, j+\tfrac{1}{2}, k\right)\right\} \\
&\quad + chzly\left\{E_x^n\left(i+\tfrac{1}{2}, j+1, k\right) - E_x^n\left(i+\tfrac{1}{2}, j, k\right)\right\}
\end{aligned}$$