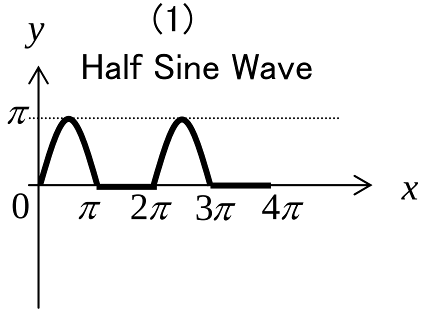
フーリエ級数展開(1)



$$
\begin{cases}
a_0 = \dfrac{1}{2\pi}\left\{\int_0^{\pi} \pi \sin x \, dx + \int_{\pi}^{2\pi} 0 \, dx\right\} \\
a_m = \dfrac{2}{2\pi}\left\{\int_0^{\pi} \pi \sin x \cos mx \, dx + \int_{\pi}^{2\pi} 0 \cos mx \, dx\right\} \\
b_n = \dfrac{2}{2\pi}\left\{\int_0^{\pi} \pi \sin x \sin nx \, dx + \int_{\pi}^{2\pi} 0 \sin nx \, dx\right\}
\end{cases}
$$

ヒント　加法定理

$$\int \sin x \cos mx \, dx = \int \frac{1}{2}\left\{\sin(1+m)x + \sin(1-m)x\right\} dx$$

$$\int \sin x \sin nx \, dx = \int -\frac{1}{2}\left\{\cos(1+n)x - \cos(1-n)x\right\} dx$$

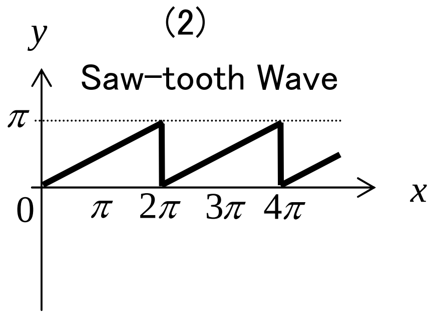
答え

$$a_0 = 1$$

$$a_1 \sim a_5 = 0, -\frac{2}{3}, 0, -\frac{2}{15}, 0$$

$$b_1 \sim b_5 = \frac{\pi}{2}, 0, 0, 0, 0$$

フーリエ級数展開(2)



$$
\begin{cases}
a_0 = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} \dfrac{x}{2}\, dx \\
a_m = \dfrac{2}{2\pi}\displaystyle\int_0^{2\pi} \dfrac{x}{2} \cos mx\, dx \\
b_n = \dfrac{2}{2\pi}\displaystyle\int_0^{2\pi} \dfrac{x}{2} \sin nx\, dx
\end{cases}
$$

ヒント　部分積分

$$\int x \cos mx\, dx = \int x \left(\frac{1}{m} \sin mx \right)' dx = \left[\frac{x}{m} \sin mx \right] - \int \frac{1}{m} \sin mx\, dx + C$$

$$\int x \sin nx\, dx = \int x \left(-\frac{1}{n} \cos nx \right)' dx = \left[-\frac{x}{n} \cos nx \right] - \int -\frac{1}{n} \cos nx\, dx + C$$

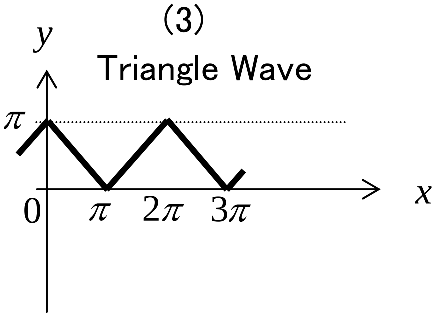
答え

$$a_0 = \frac{\pi}{2}$$

$$a_1 \sim a_5 = 0, 0, 0, 0, 0$$

$$b_1 \sim b_5 = -1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, -\frac{1}{5}$$

フーリエ級数展開(3)



$$\begin{cases} a_0 = \dfrac{1}{2\pi}\left\{\int_{-\pi}^{0}(x+\pi)\,dx + \int_{0}^{\pi}(-x+\pi)\,dx\right\} \\ a_m = \dfrac{2}{2\pi}\left\{\int_{-\pi}^{0}(x+\pi)\cos mx\,dx + \int_{0}^{\pi}(-x+\pi)\cos mx\,dx\right\} \\ b_n = \dfrac{2}{2\pi}\left\{\int_{-\pi}^{0}(x+\pi)\sin nx\,dx + \int_{0}^{\pi}(-x+\pi)\sin nx\,dx\right\} \end{cases}$$

ヒント　部分積分

$$\int (x+\pi)\cos mx\,dx = \int (x+\pi)\left(\frac{1}{m}\sin mx\right)' dx = \left[(x+\pi)\frac{1}{m}\sin mx\right] - \int \frac{1}{m}\sin mx\,dx + C$$

$$\int (x+\pi)\sin nx\,dx = \int (x+\pi)\left(-\frac{1}{n}\cos nx\right)' dx = \left[(x+\pi)\left(-\frac{1}{n}\cos nx\right)\right] - \int -\frac{1}{n}\cos nx\,dx + C$$

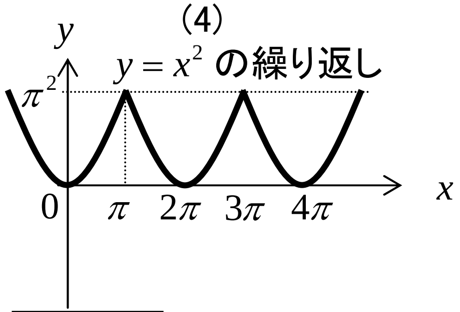
答え

$$a_0 = \frac{\pi}{2}$$

$$a_1 \sim a_5 = \frac{4}{\pi}, 0, \frac{4}{9\pi}, 0, \frac{4}{25\pi}$$

$$b_1 \sim b_5 = 0, 0, 0, 0, 0$$

フーリエ級数展開(4)



$$\begin{cases} a_0 = \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} x^2\, dx \\ a_m = \dfrac{2}{2\pi}\displaystyle\int_{-\pi}^{\pi} x^2 \cos mx\, dx \\ b_n = \dfrac{2}{2\pi}\displaystyle\int_{-\pi}^{\pi} x^2 \sin nx\, dx \end{cases}$$

ヒント　部分積分を2回実行

$$\int x^2 \cos mx\, dx = \int x^2 \left(\frac{1}{m}\sin mx\right)' dx = \left[\frac{x^2}{m}\sin mx\right] - \underbrace{\int \frac{2x}{m}\sin mx\, dx}_{} + C$$

$$\overbrace{\int \frac{2x}{m}\sin mx\, dx}^{} = \frac{2}{m}\int x\left(-\frac{1}{m}\cos mx\right)' dx = \frac{2}{m}\left[-\frac{x}{m}\cos mx\right] - \frac{2}{m}\int -\frac{1}{m}\cos mx\, dx + C$$

$$\therefore \int x^2 \cos mx\, dx = \left[\frac{x^2}{m}\sin mx\right] + \frac{2}{m}\left[\frac{x}{m}\cos mx\right] - \frac{2}{m}\int \frac{1}{m}\cos mx\, dx + C$$

答え

$$a_0 = \frac{\pi^2}{3}$$

$$a_1 \sim a_5 = -4, 1, -\frac{4}{9}, \frac{1}{4}, -\frac{4}{25}$$

$$b_1 \sim b_5 = 0, 0, 0, 0, 0$$