

特性インピーダンス

1st 2011/04/22
Lst 2023/01/20

インピーダンスの種類

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

媒質の固有インピーダンス [Ω]
Intrinsic impedance

$$Z_w = \frac{E_t}{H_t}$$

w : wave
t : transverse

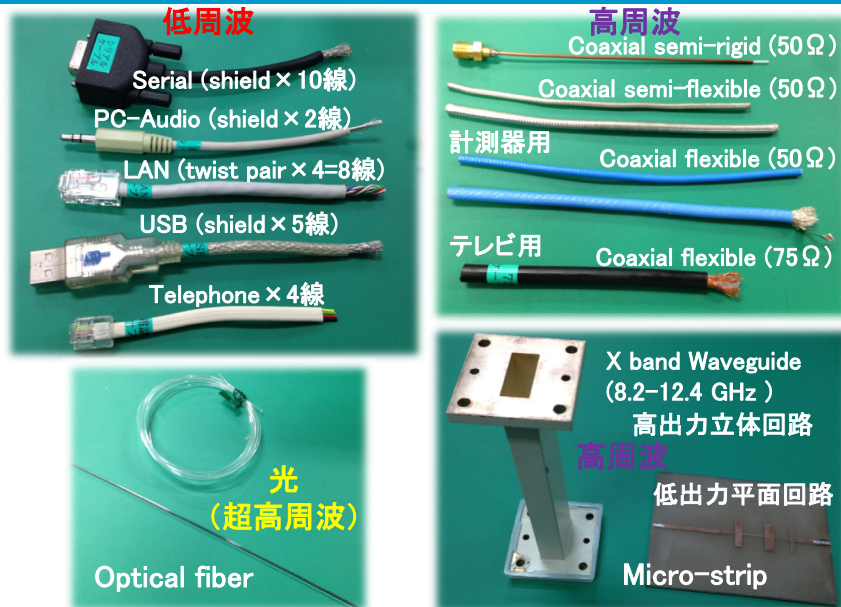
波動インピーダンス [Ω]
Wave impedance
 $Z_{TEM} = \eta \neq Z_{TE}, Z_{TM}$
(平面波では η に等しい)

$$Z_0 = \frac{V}{I} = \sqrt{\frac{L}{C}}$$

特性インピーダンス [Ω]
Characteristic impedance

D. M. Pozar, Microwave engineering 2nd, p.167, Wiley

種々の伝送線路



種々のTEMモード伝送線

<p>マイクロストリップ (MSL)</p>	<p>ストリップ</p>	<p>接地線</p>
$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln \left(\frac{8d}{W} + \frac{W}{4d} \right)$ $Z_0 = \frac{120\pi}{\sqrt{\epsilon_r}} \ln \left(\frac{d}{W} + 1.444 \right)$ <p>for $\frac{W}{d} \leq 1$ for $\frac{W}{d} \geq 1$</p>	$Z_0 = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{d}{W + 0.441d}$	$Z_0 = \frac{\eta_0}{2\pi} \sqrt{\frac{\mu_r}{\epsilon_r}} \ln \frac{2d}{a}$
<p>平行平板</p>	<p>同軸線</p> <p>単位長さあたりのCとLが求まれば、TEMモード伝送線路の特性インピーダンスは求まる</p>	<p>平行線</p>
$Z_0 = \eta_0 \sqrt{\frac{\mu_r}{\epsilon_r}} \frac{d}{W}$	$Z_0 = \frac{\eta_0}{2\pi} \sqrt{\frac{\mu_r}{\epsilon_r}} \ln \frac{b}{a}$	$Z_0 = \frac{\eta_0}{\pi} \sqrt{\frac{\mu_r}{\epsilon_r}} \ln \frac{d}{a}$

Pozar, "Microwave Engineering, 3rd," p.143-146, John Wiley & Sons 森, "マイクロウェーブ技術入門講座 基礎編," p.14, CQ出版, 2003.

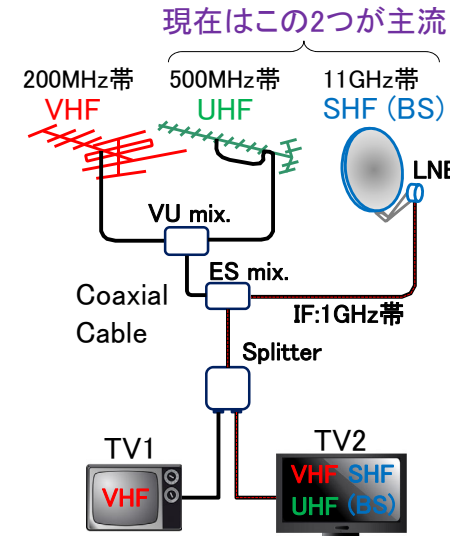
伝送線路への置き換え

分類	分布定数モデル	特性インピーダンス
無損失		$Z_0 = \sqrt{\frac{L}{C}}$
誘電損失 または 導電損失		$Z_0 = \sqrt{\frac{j\omega L}{G + j\omega C}}$
磁気導電損失 または 磁性損失		$Z_0 = \sqrt{\frac{R + j\omega L}{j\omega C}}$
一般損失 (誘電・磁性損 両方)		$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$

ただし、
 L: 単位長さあたりのインダクタンス [H/m]
 R: 単位長さあたりのレジスタンス [Ω/m]
 C: 単位長さあたりのキャパシタンス [F/m]
 G: 単位長さあたりのコンダクタンス [S/m]

導体の表皮効果はRの周波数特性R(ω)に入る。

同軸ケーブルの利用例



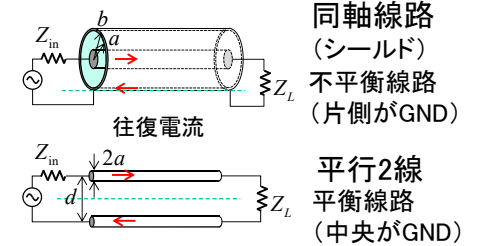
現在はこの2つが主流

地上波・衛星波テレビ用

$$\begin{aligned} \epsilon_r &= 1.0, \mu_r \approx 1.0 \\ b/a &= 3.6 \\ Z_0 &\approx 75 \Omega \end{aligned}$$

計測器用(オシロ・スペアナ等)

$$\begin{aligned} \epsilon_r &= 2.2, \mu_r \approx 1.0 \\ b/a &= 3.6 \\ Z_0 &\approx 50 \Omega \end{aligned}$$



http://www.maspro.co.jp/contact/pro/pro_02.html

同軸線路の特性インピーダンス

ガウスの法則より

If Q [C] is charged in the inner conductor, in the case of $a < r < b$
 $\oint_S \vec{D} \cdot d\vec{r} = Q$
 $\Rightarrow \oint_S D dr = Q \Rightarrow D \oint_S dr = Q \Rightarrow D 2\pi r l = Q$

$$\therefore D = \frac{Q}{2\pi r l} \quad \dots(1) \quad \left[E = \frac{D}{\epsilon} = \frac{Q}{2\pi \epsilon r l} \right] \quad \dots(1')$$

Then the potential difference V is,

$$V = - \int_a^b E dr = - \int_a^b \frac{Q}{2\pi \epsilon r l} dr = - \frac{Q}{2\pi \epsilon l} [\ln r]_a^b = \frac{Q}{2\pi \epsilon l} (\ln b - \ln a)$$

$$\therefore Q = \frac{2\pi \epsilon l V}{\ln \frac{b}{a}} \quad \dots(2)$$

Substitute (2) to (1) produces

$$E = \frac{Q}{2\pi \epsilon r l} = \frac{1}{2\pi \epsilon r l} \frac{2\pi \epsilon l V}{\ln \frac{b}{a}} = \frac{V}{r \ln \frac{b}{a}}$$

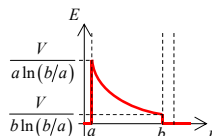
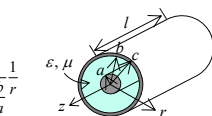
In the case of $r < a$ and $b < r$

$$E = 0$$

From equation (2)

$$C = \frac{Q}{V} = \frac{2\pi \epsilon l}{\ln \frac{b}{a}} \quad [F]$$

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アンペアの法則より

(i) In the case of $r < a$

$$\oint_C \vec{H} \cdot d\vec{l} = \oint_C H dl = \frac{\pi r^2 I}{\pi a^2}$$

$$\Rightarrow H 2\pi r = \frac{r^2 I}{a^2}$$

$$\therefore H_1 = \frac{I r}{2\pi a^2}$$

(ii) In the case of $a < r < b$

$$\oint_C \vec{H} \cdot d\vec{l} = \oint_C H dl = I$$

$$\Rightarrow H 2\pi r = I$$

$$\therefore H_2 = \frac{I}{2\pi r} \quad \dots(3)$$

(iii) In the case of $b < r < c$

$$\oint_C \vec{H} \cdot d\vec{l} = \oint_C H dl = I - \frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2} I$$

$$\Rightarrow H 2\pi r = \frac{c^2 - r^2}{c^2 - b^2} I$$

$$\therefore H_3 = \frac{I}{2\pi r} \frac{c^2 - r^2}{c^2 - b^2}$$

(iv) In the case of $r > c$

$$\oint_C \vec{H} \cdot d\vec{l} = \oint_C H dl = I - I = 0$$

$$\Rightarrow H 2\pi r = 0$$

$$\therefore H_4 = 0$$

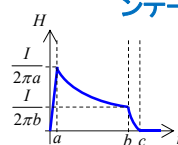
From equation (3)

$$\varphi = \int_a^b B_z dr = \int_a^b \mu H dr = \frac{\mu I}{2\pi} [\ln r]_a^b$$

$$\therefore \varphi = \frac{\mu I}{2\pi} \ln \frac{b}{a} \quad [Wb]$$

$$L = \frac{\varphi}{I} = \frac{\mu}{2\pi} \ln \frac{b}{a} \quad [H]$$

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特性インピーダンス

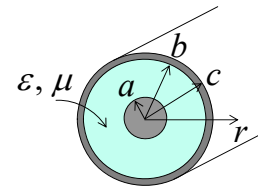
$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\frac{\mu}{2\pi} \ln \frac{b}{a}}{\frac{2\pi \epsilon l}{\ln \frac{b}{a}}}} = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln \frac{b}{a}$$

$$\therefore Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln \frac{b}{a} \quad \text{電磁波の導入テーマ}$$

同軸線路の特性インピーダンスと損失

特性インピーダンス

$$Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln \left(\frac{b}{a} \right) = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{1}{\sqrt{\epsilon_r}} \ln \left(\frac{b}{a} \right) = \frac{59.9586}{\sqrt{\epsilon_r}} \ln \left(\frac{b}{a} \right) = \frac{138.061}{\sqrt{\epsilon_r}} \log \left(\frac{b}{a} \right) \quad (1)$$



a, b, ε の値によって Z₀ を自由に設計できるが、

底変換の公式

$$\frac{\log_{\alpha} x}{y} = \frac{\log_{\alpha} \beta \times \log_{\beta} x}{z} \Rightarrow x = \alpha^y, x = \beta^z, \beta = \alpha^c$$

$$\Rightarrow x = \alpha^y = \beta^z = (\alpha^c)^z = \alpha^{cz} \Rightarrow \therefore y = cz$$

α → e, x → b/a, β → 10 に置き換えると、

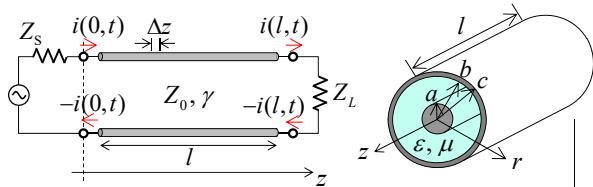
$$\therefore \log_e \left(\frac{b}{a} \right) = \log_e 10 \times \log_{10} \left(\frac{b}{a} \right) = 2.3026 \times \log_{10} \left(\frac{b}{a} \right)$$

導体損失 (TEMモード)

$$\alpha_{cTEM} = \frac{1}{2} \frac{R_s}{\eta \ln b/a} \left(\frac{1}{a} + \frac{1}{b} \right) = \frac{1}{2} \frac{\sqrt{\omega \mu_0 / 2\sigma}}{\eta \ln b/a} \frac{1}{b} \left(\frac{b}{a} + 1 \right) \propto \frac{b/a + 1}{\ln b/a} = \frac{x + 1}{\ln x} = f(x) \quad (2)$$

※ α=f(x) が最小となる x=b/a の値は 3.5911

同軸線路の導体損失



損失電力は

$$P_{Loss} = \frac{\rho l}{2\pi\delta_s} \frac{P}{Z_0} \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$= P \frac{\rho l}{2\pi\delta_s} \frac{2\pi}{\eta} \frac{1}{\ln b/a} \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$= P \frac{\rho l}{\delta_s \eta} \frac{1}{\ln b/a} \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$= P \frac{\rho l}{\eta \sqrt{2/\omega\mu\sigma}} \frac{1}{\ln b/a} \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$= P \frac{\rho l \sqrt{\omega\mu\sigma/2}}{\eta \ln b/a} \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$= P \frac{l \sqrt{\omega\mu/2\sigma}}{\eta \ln b/a} \left(\frac{1}{a} + \frac{1}{b} \right) \quad [W] \quad (6)$$

回路パラメータによる導体損失の計算

$$I = \sqrt{\frac{P}{Z_0}} \quad [A], \quad Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right) \quad [\Omega] \quad (1)$$

$$P = VI = Z_0 I^2 \quad [W] \quad (2)$$

$$P_{Loss} = (R_{ic} + R_{oc}) I^2 = (R_{ic} + R_{oc}) \frac{P}{Z_0} \quad (3)$$

$$R_{ic} = \rho \frac{l}{2\pi a \delta_s} \quad [\Omega], \quad R_{oc} = \rho \frac{l}{2\pi b \delta_s} \quad [\Omega] \quad (4)$$

$$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}} \quad \text{表皮厚み} \quad (5)$$

電子回路部品活用マニュアル第2集, トランジスタ技術 SPECIAL No.97, pp.128-131, CQ出版社

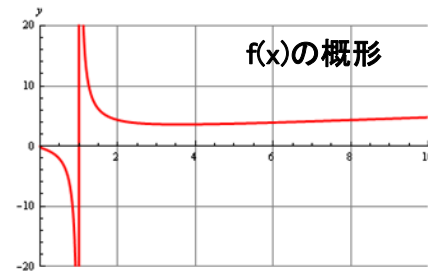
導体損が最小となるb/a比

$$f(x) = \frac{x+1}{\ln x} \quad (1)$$

$$f'(x) = \left(\frac{x+1}{\ln x} \right)' = -\frac{x+1}{x \ln x^2} + \frac{1}{\ln x} \quad (2)$$

```
f[x_] := (x + 1) / Log[x]
```

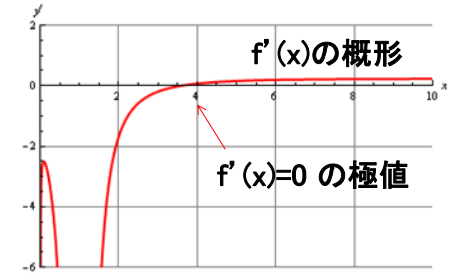
```
Plot[f[x], {x, 0, 10}, AxesLabel -> {x, y}, PlotStyle -> {Red}]
```



```
D[f[x], x]
```

```
- (1 + x) / (x Log[x]^2) + 1 / Log[x]
```

```
Plot[- (1 + x) / (x Log[x]^2) + 1 / Log[x], {x, 0, 10}, AxesLabel -> {x, y'}
```

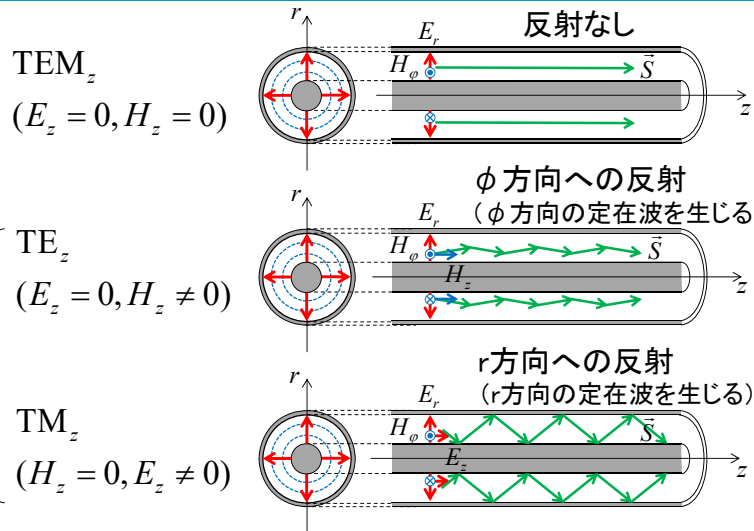


```
FindRoot[- (1 + x) / (x Log[x]^2) + 1 / Log[x] == 0, {x, 2}]
```

{x -> 3.59112}

数学解析ソフトMathematica10, ニュートン法

同軸線路の高次モード

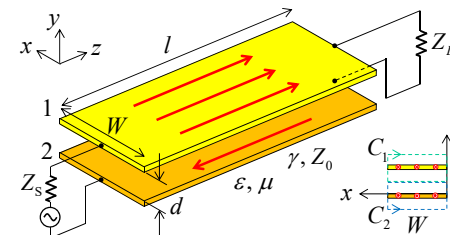


カットオフ周波数

最低次の高次モードが伝搬開始する周波数

$$f_c = \frac{2c}{\pi \sqrt{\epsilon_r} (D+d)}$$

平行平板の特性インピーダンス



アンペアの法則より

$$\oint_{C_1} \vec{H} \cdot d\vec{l} = I \quad \oint_{C_2} \vec{H} \cdot d\vec{l} = I$$

$$\Rightarrow H_2 W = I \quad \Rightarrow H_2 W = I$$

$$\Rightarrow H_1 = \frac{I}{2W} \quad (5) \quad \Rightarrow H_2 = \frac{I}{2W} \quad (6)$$

$$\vec{H} = \vec{H}_1 + \vec{H}_2 = \frac{I}{2W} \hat{x} + \frac{I}{2W} \hat{x} = \frac{I}{W} \hat{x} \quad (7)$$

$$\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I}{W} \hat{x} \quad (8)$$

$$\Phi = \oint_S \vec{B} \cdot d\vec{s} = \int_{y=0}^1 \int_{z=0}^1 \frac{\mu_0 I}{W} dz dy = \frac{\mu_0 I}{W} d \quad (9)$$

$$L = \frac{\Phi}{I} = \frac{\mu_0 I d}{I} = \mu_0 \frac{d}{W} \quad [H/m] \quad (10)$$

特性インピーダンスは

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu_0 \mu_r d / W}{\epsilon_0 \epsilon_r W / d}} = \sqrt{\frac{\mu_r}{\epsilon_r}} \frac{d}{W} \quad [\Omega] \quad (11)$$

ガウスの法則より

$$\oint_S \vec{D} \cdot d\vec{s} = Q \quad V = - \int_{y=0}^d \vec{E} \cdot d\vec{l}$$

$$\Rightarrow DW = Q \quad = - \int_{y=0}^d \vec{E} \cdot d\vec{y}$$

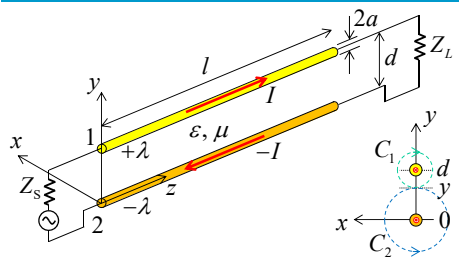
$$\Rightarrow D = \frac{Q}{W} \quad (1) \quad \Rightarrow V = \frac{Qd}{\epsilon_0 \epsilon_r W} \quad (2)$$

$$\left\{ \vec{E} = \frac{\vec{D}}{\epsilon} = \frac{\vec{D}}{\epsilon_0 \epsilon_r} = \frac{Q}{\epsilon_0 \epsilon_r W} (-\hat{y}) \right. \quad (3)$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{\epsilon_0 \epsilon_r W}} = \frac{\epsilon_0 \epsilon_r W}{d} \quad [F/m] \quad (4)$$

鈴木, ``デジタル回路のEMC設計技術入門,`` pp. 26-29, 日刊工業新聞社, 2011

平行線路の特性インピーダンス



アンペアの法則より

$$\vec{H} = \vec{H}_1 + \vec{H}_2 \quad \text{アンペアの法則より}$$

$$\vec{H} = \frac{I}{2\pi(d-y)} \hat{x} + \frac{I}{2\pi y} \hat{x} \quad (4)$$

$$\varphi = \int_S \vec{B} \cdot d\vec{s} = \int_{z=0}^l \int_{y=a}^{d-a} \frac{\mu_0 I}{2\pi(d-y)} + \frac{\mu_0 I}{2\pi y} dy dz$$

$$= \frac{\mu_0 I}{2\pi} \int_{y=a}^{d-a} \frac{1}{d-y} + \frac{1}{y} dy$$

$$= \frac{\mu_0 I}{2\pi} \left(-[\ln(d-y)]_a^{d-a} + [\ln y]_a^{d-a} \right) \quad (5)$$

$$= \frac{\mu_0 I}{2\pi} \left(-\ln \frac{a}{d-a} + \ln \frac{d-a}{a} \right) = \frac{\mu_0 I}{\pi} \ln \frac{d-a}{a}$$

ガウスの法則より

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0(d-y)} (-\hat{y}) + \frac{\lambda}{2\pi\epsilon_0 y} (-\hat{y}) \quad (1)$$

$$V = -\int_{y=a}^{d-a} \vec{E} \cdot d\vec{y} = \frac{\lambda}{2\pi\epsilon_0} \int_{y=a}^{d-a} \frac{1}{d-y} + \frac{1}{y} dy$$

$$= \frac{\lambda}{2\pi\epsilon_0} \left(-[\ln(d-y)]_a^{d-a} + [\ln y]_a^{d-a} \right) \quad (2)$$

$$= \frac{\lambda}{2\pi\epsilon_0} \left(-\ln \frac{a}{d-a} + \ln \frac{d-a}{a} \right) = \frac{\lambda}{\pi\epsilon_0} \ln \frac{d-a}{a}$$

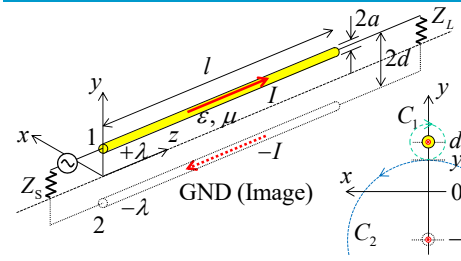
$a \ll d$ ならば $V \approx \frac{\lambda}{\pi\epsilon_0} \ln \frac{d}{a} \Rightarrow C = \frac{Q}{V} = \pi\epsilon_0 / \ln \frac{d}{a}$ (3)

特性インピーダンスは

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\frac{\mu_0}{\pi} \ln \frac{d}{a}}{\pi\epsilon_0 / \ln \frac{d}{a}}} = \frac{1}{\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \ln \frac{d}{a} \quad [\Omega] \quad (7)$$

$a \ll d$ ならば $\varphi \approx \frac{\mu_0 I}{\pi} \ln \frac{d}{a} \Rightarrow L = \frac{\varphi}{I} = \frac{\mu_0}{\pi} \ln \frac{d}{a}$ (6)

平行線路の特性インピーダンス



アンペアの法則より

$$\vec{H} = \vec{H}_1 + \vec{H}_2$$

$$\vec{H} = \frac{I}{2\pi(d-y)} \hat{x} + \frac{I}{2\pi(d+y)} \hat{x} \quad (4)$$

$$\varphi = \int_S \vec{B} \cdot d\vec{s} = \int_{z=0}^l \int_{y=0}^{d-a} \frac{\mu_0 I}{2\pi(d-y)} + \frac{\mu_0 I}{2\pi(d+y)} dy dz$$

$$= \frac{\mu_0 I}{2\pi} \int_{y=0}^{d-a} \frac{1}{d-y} + \frac{1}{d+y} dy$$

$$= \frac{\mu_0 I}{2\pi} \left(-[\ln(d-y)]_0^{d-a} + [\ln(d+y)]_0^{d-a} \right) \quad (5)$$

$$= \frac{\mu_0 I}{2\pi} \left(-\ln \frac{a}{d} + \ln \frac{2d-a}{d} \right) = \frac{\mu_0 I}{2\pi} \ln \frac{2d-a}{a}$$

ガウスの法則より

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0(d-y)} (-\hat{y}) + \frac{\lambda}{2\pi\epsilon_0(d+y)} (-\hat{y}) \quad (1)$$

$$V = -\int_{y=0}^{d-a} \vec{E} \cdot d\vec{y} = \frac{\lambda}{2\pi\epsilon_0} \int_{y=0}^{d-a} \frac{1}{d-y} + \frac{1}{d+y} dy$$

$$= \frac{\lambda}{2\pi\epsilon_0} \left(-[\ln(d-y)]_0^{d-a} + [\ln(d+y)]_0^{d-a} \right) \quad (2)$$

$$= \frac{\lambda}{2\pi\epsilon_0} \left(-\ln \frac{a}{d} + \ln \frac{2d-a}{d} \right) = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{2d-a}{a}$$

$a \ll 2d$ ならば $V \approx \frac{\lambda}{2\pi\epsilon_0} \ln \frac{2d}{a} \Rightarrow C = \frac{Q}{V} = 2\pi\epsilon_0 / \ln \frac{2d}{a}$ (3)

特性インピーダンスは

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\frac{\mu_0}{2\pi} \ln \frac{2d}{a}}{2\pi\epsilon_0 / \ln \frac{2d}{a}}} = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \ln \frac{2d}{a} \quad [\Omega] \quad (7)$$

$a \ll d$ ならば $\varphi \approx \frac{\mu_0 I}{2\pi} \ln \frac{2d}{a} \Rightarrow L = \frac{\varphi}{I} = \frac{\mu_0}{2\pi} \ln \frac{2d}{a}$ (6)

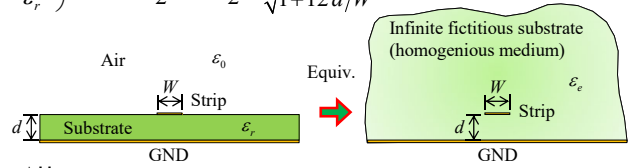
MSLの特性インピーダンス

$$Z_0 = \begin{cases} \frac{60}{\sqrt{\epsilon_e}} \ln \left(\frac{8d}{W} + \frac{W}{4d} \right) & \text{for } W/d \leq 1 \\ \frac{120\pi}{\sqrt{\epsilon_e} [W/d + 1.393 + 0.667 \ln(W/d + 1.444)]} & \text{for } W/d \geq 1. \end{cases} \quad (1)$$

$$\frac{W}{d} = \begin{cases} \frac{8e^A}{e^{2A} - 2} & \text{for } W/d \leq 2 \\ \frac{2}{\pi} \left[B - 1 - \ln(2B-1) + \frac{\epsilon_r - 1}{2\epsilon_r} \left\{ \ln(B-1) + 0.39 - \frac{0.61}{\epsilon_r} \right\} \right] & \text{for } W/d \geq 2, \end{cases} \quad (2)$$

$$A = \frac{Z_0}{60} \sqrt{\frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{\epsilon_r} \left(0.23 + \frac{0.11}{\epsilon_r} \right)}, \quad \epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12d/W}} \quad (3)$$

$$B = \frac{377\pi}{2Z_0\sqrt{\epsilon_r}} \quad (4)$$



特性インピーダンスの導出(別解)

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{\sqrt{LC}}{C} = \frac{1}{v_p} \frac{1}{C} = \frac{\sqrt{\epsilon_{eff}}}{c} \frac{1}{C} \quad \because v_p = \frac{1}{\sqrt{LC}} = \frac{c}{\sqrt{\epsilon_{eff}}} \quad (1)$$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{L}{\sqrt{LC}} = v_p L = \frac{c}{\sqrt{\epsilon_{eff}}} L \quad \because v_p = \frac{1}{\sqrt{LC}} = \frac{c}{\sqrt{\epsilon_{eff}}} \quad (2)$$

位相速度 v_p の分かっている伝送線路の特性インピーダンスは、 C または L どちらかが求まれば導出できる。