フーリエ解析公式集

v1.5 Sep.2021　　　番号:____________________　氏名:________________________

1. 関数の掛け合わせの性質

偶関数 × 偶関数 = 偶関数　(1)

奇関数 × 奇関数 = 偶関数　(2)

偶関数 × 奇関数 = 奇関数　(3)

2. 三角関数の合成

$$a_n \cos n\omega_0 t + b_n \sin n\omega_0 t = c_n \{\cos(n\omega_0 t - \theta_n)\} \tag{4}$$

$$c_n = \sqrt{{a_n}^2 + {b_n}^2} \tag{5}$$

$$\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right) \tag{6}$$

3. インパルス関数（デルタ関数）の性質

$$\int_{-\infty}^{\infty} \delta(t)dt = 1 \tag{7}$$

$$\int_{-\infty}^{\infty} \delta(t)f(t)dt = f(0) \tag{8}$$

$$\int_{-\infty}^{\infty} \delta(t-t_0)f(t)dt = f(t_0) \tag{9}$$

4. フーリエ級数

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t), \quad \omega_0 = \frac{2\pi}{T} \tag{10}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(n\omega_0 t)\,dt, \quad n = 0, 1, 2, \cdots \tag{11}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(n\omega_0 t)\,dt, \quad n = 1, 2, \cdots \tag{12}$$

5. 複素フーリエ級数

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T} \tag{13}$$

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_0 t}dt, \quad n = 0,\ \pm1,\ \pm2, \cdots \tag{14}$$

6. フーリエ変換

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \tag{15}$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega \tag{16}$$

または，$\omega = 2\pi f$ より $d\omega = 2\pi df$ を代入して

$$F(f) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi ft}dt \tag{17}$$

$$f(t) = \int_{-\infty}^{\infty} F(f)e^{j2\pi ft}df \tag{18}$$

7. フーリエ変換の性質

$$\mathcal{F}[a_1 f_1(t) + a_2 f_2(t)] = a_1 F_1(\omega) + a_2 F_2(\omega) \qquad \text{線形則} \tag{19}$$

$$\mathcal{F}[f(at)] = \frac{1}{|a|}F\left(\frac{\omega}{a}\right) \qquad \text{スケーリング則} \tag{20}$$

$$\mathcal{F}[f(t-t_0)] = F(\omega)e^{-j\omega t_0} \qquad \text{時間推移則} \tag{21}$$

$$\mathcal{F}\left[f(t)e^{j\omega_0 t}\right] = F(\omega-\omega_0) \qquad \text{周波数推移則} \tag{22}$$

$$\mathcal{F}\left[f^{(n)}(t)\right] = (j\omega)^n F(\omega) \qquad \text{時間微分則} \tag{23}$$

$$\mathcal{F}[-jt\,f(t)] = \frac{dF(\omega)}{d\omega} \qquad \text{周波数微分則} \tag{24}$$

8. 良く使うフーリエ変換の例

$$\mathcal{F}[K\delta(t)] = K, \quad \mathcal{F}[\delta(t)] = 1 \tag{25}$$

$$\mathcal{F}[\delta(t-t_0)] = e^{-j\omega t_0} = 1\angle -\omega t_0 \tag{26}$$

$$\mathcal{F}[K] = K2\pi\delta(\omega) = K\delta(f), \quad \mathcal{F}[1] = 2\pi\delta(\omega) = \delta(f) \tag{27}$$

$$\mathcal{F}\left[e^{j\omega_0 t}\right] = 2\pi\delta(\omega-\omega_0) = \delta(f-f_0) \tag{28}$$

$$\mathcal{F}[\cos\omega_0 t] = \pi\delta(\omega+\omega_0)+\pi\delta(\omega-\omega_0) = \frac{1}{2}\delta(f+f_0)+\frac{1}{2}\delta(f-f_0) \tag{29}$$

$$\mathcal{F}[\sin\omega_0 t] = j\pi\delta(\omega+\omega_0)-j\pi\delta(\omega-\omega_0) = \frac{j}{2}\delta(f+f_0)-\frac{j}{2}\delta(f-f_0) \tag{30}$$

$$\mathcal{F}[u(t)] = \pi\delta(\omega) + \frac{1}{j\omega}, \quad (t>0 \text{ で } u(t)=1, t<0 \text{ で } u(t)=0) \tag{31}$$

$$\mathcal{F}[\mathrm{sgn}(t)] = \frac{2}{j\omega}, \quad (t>0 \text{ で } \mathrm{sgn}(t)=1, t<0 \text{ で } \mathrm{sgn}(t)=-1) \tag{32}$$

9. 周期関数のフーリエ変換

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T} \tag{33}$$

$$\mathcal{F}[f(t)] = \sum_{n=-\infty}^{\infty} c_n 2\pi\delta(\omega-n\omega_0) = \sum_{n=-\infty}^{\infty} c_n\delta(f-nf_0) \tag{34}$$

10. 単位インパルス列のフーリエ変換

$$\delta_T(t) = \sum_{n=-\infty}^{\infty}\delta(t-nT) = \frac{1}{T}\sum_{n=-\infty}^{\infty} e^{jn\omega_0 t} \tag{35}$$

$$\mathcal{F}[\delta_T(t)] = \frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0) = \frac{1}{T}\sum_{n=-\infty}^{\infty}\delta(f-nf_0) \tag{36}$$

11. たたみ込み

$$f_1(t)*f_2(t) = \int_{-\infty}^{\infty} f_1(t)f_2(t-x)dx \tag{37}$$

$$f_1(t)*f_2(t) = f_2(t)*f_1(t) \qquad \text{可換則} \tag{38}$$

$$f_1(t)*\delta(t) = f(t) \tag{39}$$

$$f_1(t)*\delta(t-T) = f(t-T) \tag{40}$$

$$f_1(t-t_1)*\delta(t-t_2) = f(t-t_1-t_2) \tag{41}$$

$$\mathcal{F}[f_1(t)*f_2(t)] = F_1(\omega)F_2(\omega) \qquad \text{時間たたみ込み} \tag{42}$$

$$\mathcal{F}[f_1(t)f_2(t)] = \frac{1}{2\pi}F_1(\omega)*F_2(\omega) \qquad \text{周波数たたみ込み} \tag{43}$$

12. 離散フーリエ変換

$$H\left(\frac{n}{NT}\right) = \sum_{k=0}^{N-1} h(kT)e^{-j2\pi nk/N} \quad (n = 0, 1, 2, \cdots, N-1) \tag{44}$$

$$\left|H\left(\frac{n}{NT}\right)\right| = \sqrt{{a_n}^2 + {b_n}^2} \tag{45}$$

$$\angle H\left(\frac{n}{NT}\right) = \tan^{-1}\left(\frac{-b_n}{a_n}\right) \tag{46}$$

$$a_n = \sum_{k=0}^{N-1} h(kT)\cos(2\pi nk/N) \tag{47}$$

$$b_n = \sum_{k=0}^{N-1} h(kT)\sin(2\pi nk/N) \tag{48}$$

13. 高速フーリエ変換